TAKING ACCOUNT OF THE EFFECT OF TURBULENT PULSATIONS ON THE THERMAL RADIATION OF A MEDIUM IN A QUADRATIC APPROXIMATION

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An expression is obtained for the mean heat-radiation intensity of a plane layer, taking account of turbulent pulsations.

In calculating the radiation of volumes filled with turbulent gas or plasma, it is usual to use a field of thermodynamic parameters averaged over a sufficiently large time interval. However, the local temperature and concentration values of the radiating components in turbulent flows pulsate irregularly about the mean value. Because the emissivity depends nonlinearly on the temperature, the time-averaged radiation intensity (as recorded on an instrument) may differ from the value of the radiation determined from the mean values of the temperature and concentration fields.

The effect of turbulent pulsations on the thermal radiation was considered earlier in [1, 2]. In [3, 4], this problem was discussed in connection with estimates of the accuracy of optical temperature-measurement methods. In [1], a transfer equation was obtained for the case of optically thin pulsations defined by the relation $\varkappa l \ll 1$ ($\varkappa$ is the absorption coefficient, l is the characteristic dimension of the large turbulent bodies). The opposite limiting case $\varkappa l \gg 1$ was considered essentially without paying attention to absorption pulsations. For the intermediate case $\varkappa l \sim 1$, some interpolation of the results obtained for $\varkappa l \gg 1$ and $\varkappa l \ll 1$ was proposed. Absorption pulsations were taken into account in a linear approximation and pulsations of the Planck function in a quadratic approximation with respect to the temperature pulsations.

In [2], this problem was solved using functional averaging. However, only the case of a combined normal distribution of the Planck-function and absorption-coefficient pulsations was considered in [2]; this is valid for small fluctuation amplitudes, and is in fact true only for a linear dependence of the Planck function and the absorption coefficient on the temperature fluctuations.

The effect of turbulent pulsations on the radiation would be expected to be at its greatest in the blue region of the spectrum, which is characterized by a stronger than quadratic dependence of the Planck function on the temperature fluctuations. In [5], the effect of turbulent pulsations on the radiation transfer was considered in media with an arbitrary spatial distribution of the mean temperature and an exponential dependence of the Planck function on the temperature pulsations. The dependence of the absorption coefficient on the pulsating temperature component was assumed to be linear in [5].

In the present work, the case of a quadratic dependence of the absorption coefficient on the temperature pulsations is considered, allowing larger-amplitude absorption-coefficient pulsations than in the linear approximation to be taken into account. Because of the mathematical difficulties involved, calculation is restricted to the case of a statistically homogeneous layer.

The instantaneous value of the radiation intensity corresponding to a fixed temperature and concentration dependence along the line of sight is expressed as follows

$$I = \int_0^L i(x)\,dx, \tag{1}$$

where

$$i(x) = B(x)\,\varkappa(x)\exp\left\{-\int_x^L \varkappa(y)\,dy\right\}, \tag{2}$$

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0 and L are the limits of the radiating object; B(x) is the Planck function; $\varkappa(x)$ is the absorption coefficient. Because of the temperature and concentration pulsations in the direction of observation, the Planck function and the absorption coefficient oscillate in a disorderly manner around their mean values. In view of the ergodicity hypothesis, time averaging of the radiation in Eq. (1) may be replaced by averaging over the ensemble of radiating objects. Below, all the means are understood to be means over the ensemble.

Consideration will be restricted to the blue spectral region. The dependence of the Planck function on the temperature pulsations is then written in the form

$$B(x) = B_0 \exp[\beta t(x)], \tag{3}$$

where B_0 is the Planck function for the mean temperature $\langle T \rangle$, constant along the layer, while

$$\beta = h\nu / k \langle T \rangle ; \tag{4}$$

$$t(x) = [T(x) - \langle T \rangle] / \langle T \rangle ; \tag{5}$$

T(x) is the instantaneous temperature at point x. The absorption coefficient depends on the temperature and concentration. The dependence on the temperature is usually the stronger. In the present work, the case in which the pulsating component of the absorption coefficient depends solely on the temperature is considered in the following manner:

$$\varkappa(x) = \kappa[1 + \alpha_1 t(x) + \alpha_2 t^2(x)], \tag{6}$$

where k is the absorption coefficient determined from the mean temperature; α_1 and α_2 are dimensionless coefficients which do not depend on the spatial coordinate.

The result of averaging Eq. (1) depends on the form of the multidimensional temperature distribution function t(x). However, there is at present practically no information on the distribution functions for the pulsating parameters in turbulent media [1, 6]. Therefore, the simplest assumption regarding the distribution function t(x) will be made – specifically, that t(x) is a Gaussian random process with a mean value $\langle t(x) \rangle = 0$.

Noting that the mean value of the integral in Eq. (1) is equal to the integral of the mean value of the integrand in Eq. (2), consideration may pass to the calculation of the mean value of the functional in Eq. (2) for a Gaussian process, taking into account that the temperature dependence of the Planck function and the absorption coefficient is determined by Eqs. (3) and (6):

$$\langle i(x) \rangle = B_0 \langle \exp[\beta t(x)] \frac{d}{dx} \tau[t(y); x] \rangle , \tag{7}$$

where

$$\tau[t(y);\ x] = \exp\left\{-\int_x^L \kappa[1 + \alpha_1 t(y) + \alpha_2 t^2(y)]\, dy\right\}. \tag{8}$$

In the case of a Gaussian process, according to [7], the mean value $\langle i(x) \rangle$ may be written in the form

$$\begin{aligned} &\langle \exp[\beta t(x)] \frac{d}{dx} \{\tau[t(y);\ x]\} \rangle = \\ &= \langle \exp\left\{\beta\left[t(x) + \sigma^2 \int_x^L R(x,\ y) \frac{\delta}{\delta\eta(y)}\, dy\right]\right\} \rangle \frac{d}{dx} \langle \tau[t(y) + \eta(y);\ x] \rangle |_{\eta=0}. \end{aligned} \tag{9}$$

There $\sigma^2 R(x,\ y) \equiv \langle t(x) t(y) \rangle$ is a correlation function; σ^2 is the dispersion of the temperature pulsations; $\delta/\delta\eta(y)$ is the functional-derivative operator; $\eta(y)$ is a determined arbitrary function, which must be equal to zero in the final expression. Averaging the transmission in Eq. (8), which is the exponential of a quadratic functional, the following result is obtained [8]:

$$\langle \tau[t(y) + \eta(y) x] \rangle = \exp\left\{-\kappa(L - x) - \int_x^L \kappa[\alpha_1 \eta(y) + \alpha_2 \eta^2(y)]\, dy\right\} \exp\left\{\frac{1}{2} \sum_{m=1}^{\infty} \frac{c_m^2 \lambda_m}{1 + 2\kappa\alpha_2\sigma^2\lambda_m}\right\} \prod_{m=1}^{\infty} (1 + 2\kappa\alpha_2\sigma^2\lambda_m)^{-\frac{1}{2}}, \tag{10}$$

$$c_m = \int_x^L f(y) \varphi_m(y)\, dy, \tag{11}$$

$$f(y) = \kappa\sigma[\alpha_1 + 2\alpha_2\eta(y), \tag{12}$$

$\varphi_m(y)$ and λ_m being the eigenfunctions and eigenvalues of the integral equation

$$\lambda_m\varphi_m(y) = \int_x^L R(y, z)\varphi_m(z)\,dz. \tag{13}$$

The eigenvalues of the kernel, which is the correlation function, are positive. Estimation of the spectral radius [9] shows that the maximum eigenvalue

$$\lambda \leqslant \max_{y\in(x,\,L)} \int_x^L R(y, z)\,dz. \tag{14}$$

In particular, for a correlation function of the form

$$R(y, z) = \exp\left(-\left|\frac{y-z}{l}\right|\right), \tag{15}$$

where l is the characteristic dimension of the pulsations, Eq. (14) gives

$$\lambda \leqslant 2l\left[1 - \exp\left(-\frac{L-x}{2l}\right)\right]. \tag{16}$$

When the condition

$$2\kappa\alpha_2\sigma^2\lambda < 1 \tag{17}$$

is satisfied, the infinite products and sums in Eq. (10) may be transformed as follows:

$$\prod_{m=1}^{\infty}(1 + 2\kappa\alpha_2\sigma^2\lambda_m)^{-\frac{1}{2}} = \exp\left\{-\frac{1}{2}\sum_{m=1}^{\infty}\ln(1 + 2\kappa\alpha_2\sigma^2\lambda_m)\right\} = \exp\left\{-\frac{1}{2}\sum_{n=1}^{\infty}(-1)^{n+1}\frac{(2\kappa\alpha_2\sigma^2)^n}{n}\sum_{m=1}^{\infty}\lambda_m^n\right\}; \tag{18}$$

$$\sum_{m=1}^{\infty}\frac{c_m^2\lambda_m}{1 + 2\kappa\alpha_2\sigma^2\lambda_m} = \sum_{n=1}^{\infty}(-2\kappa\alpha_2\sigma^2)^{n-1}\sum_{m=1}^{\infty}c_m^2\lambda_m^n. \tag{19}$$

In the case of symmetric kernels, whatever the correlation functions, the sums over m may be written in the form

$$\sum_{m=1}^{\infty}\lambda_m^n = \int_x^L R^{(n)}(y, y)\,dy, \tag{20}$$

$$\sum_{m=1}^{\infty}c_m^2\lambda_m^n = \int_x^L\int_x^L f(y)f(z)R^{(n)}(y, z)\,dydz, \tag{21}$$

where $R^{(n)}$ is the iterated kernel.

Substituting Eqs. (18)-(21) into Eq. (10), and retaining only the first two terms in the sums over n, the mean transmission may be obtained. If the derivative of the transmission is then substituted into Eq. (9) and subjected to the functional-shift operator, the following expression is obtained:

$$\langle i(x)\rangle = \langle B\varkappa\rangle[1 - A(x)\exp\{-\langle\varkappa\rangle(L-x) + D(x)\}, \tag{22}$$

where

$$\langle B\varkappa\rangle = B_0\kappa\exp\left(\frac{1}{2}q^2\right)[1 + mq + pq^2 + p]; \tag{23}$$

$$\langle\varkappa\rangle = \kappa(1 + p); \tag{24}$$

$$A(x) = \Big\{2\kappa p^2\int_x^L R^2(x, y)\,dy + \kappa m^2(1 + c)\int_x^L[1 + cR(x, y)][R(x, y) - 2\kappa p\int_x^L R(x, z)R(y, z)\,dz]\,dy - \tag{25}$$

$$-p(\kappa m)^2\left[\int_x^L [1+cR(x,y)\,R(x,y)\,dy]^2\right]\ [1+mq+pq^2+p]^{-1}; \quad (25)$$

$$D(x)=-\kappa mq\int_x^L\left[R(x,y)+\frac{c}{2}R^2(x,y)\right]dy$$

$$+\frac{1}{2}(\kappa m)^2\int_x^L\int_x^L[1+cR(x,y)][1+cR(x,z)]\left\{R(y,z)\right. \quad (26)$$

$$\left.-2\kappa p\int_x^L R(y,u)\,R(z,u)\,du\right\}dydz-(\kappa p)^2\int_x^L\int_x^L R^2(y,z)\,dydz;$$

$$c=\frac{2\alpha_2\beta\sigma^2}{\alpha_1};\quad m=\alpha_1\sigma;\quad p=\alpha_2\sigma^2;\quad q=\beta\sigma. \quad (27)$$

It is readily evident that at small optical depths of one pulsation ($kl \ll 1$), A and D tend to zero linearly in kl and Eq. (22) transforms to the result given in [1]. In this case, taking account of the turbulent temperature pulsations reduces to replacing the product of Planck functions by the absorption coefficient, and replacing the absorption coefficient determined from the mean temperature by the mean value of the local emissivity and the absorption coefficient. This replacement may lead to an increase by a few times in the radiation of a statistically homogeneous layer. For example, when $\alpha_1 = \alpha_2 = 4$, $\beta = 5$, $\sigma = 0.2$, the radiation is three times larger when turbulent pulsations are taken into account than in a calculation from the mean temperature.

At small values of the parameter kl, the radiation of a homogeneous layer may be written as a series in powers of kl, Because the resulting expressions are cumbersome, only the result for an infinite layer in an approximation linear in kl will be given

$$\langle I\rangle=\frac{\langle \varkappa B\rangle}{\langle\varkappa\rangle}(1-\kappa lH), \quad (28)$$

$$H=\frac{mq}{1+p}+\frac{1}{2}pq^2+\frac{mq(4p-m^2)+p^2q[2q+2m+pq+m(1+q^2)]}{(1+p)(1+mq+pq^2+p)}. \quad (29)$$

Under the condition $4p > m^2$, which means that the instantaneous values of the absorption coefficient are nonnegative, $H > 0$. Therefore, taking into account that kl is finite reduces the radiation of the layer.

To estimate the effect of kl on the radiation of a layer of finite thickness for sufficiently large kl. Eq. (22) was numerically integrated, taking account of Eqs. (23)-(26) with the correlation function in Eq. (15). The results of the calculation show that taking account of A and D in Eq. (22) always reduces the radiation. This reduction becomes significant when kl is sufficiently large and when the pulsation amplitude of the absorption coefficient and the Planck function becomes comparable with their mean values, i.e., at sufficiently large values of α_1, α_2, and β. The physical reason for this reduction is the negative correlation between the local emissivity and the transmission. In Fig. 1, as an example, the radiation of the layer as a function of its optical depth is shown for the parameter values $\alpha_1 = \alpha_2 = 4$, $\beta = 5$, $\sigma = 0.2$, ensuring sufficiently large fluctuations of the Planck function and the absorption coefficient. The upper curves are obtained in the approximation of optically thin pulsations [1] and the lower without taking account of the pulsations. It is evident from Fig. 1 that calculations using Eq. (22) are little different from [1] right up to values $kl \sim 0.5$. When $kl = 1$, the optically thin pulsation approximation gives a threefold increase in the radiation, whereas calculation from Eq. (22) gives an approximately twofold increase.

Finally, an order-of-magnitude estimate will be made of the terms omitted from Eqs. (18) and (19), which are of third order and higher in λ_m. The relative contribution of the omitted terms $\sim a^2 = (2k\alpha_2\sigma^2\lambda)^2$. Using Eq. (16) for λ and noting that the region $L - x \sim k^{-1}$ makes the main contribution to the radiation of the layer, it is found that for this region

$$a=4\kappa l\alpha_2\sigma^2\left[1-\exp\left(-\frac{1}{2\kappa l}\right)\right]. \quad (30)$$

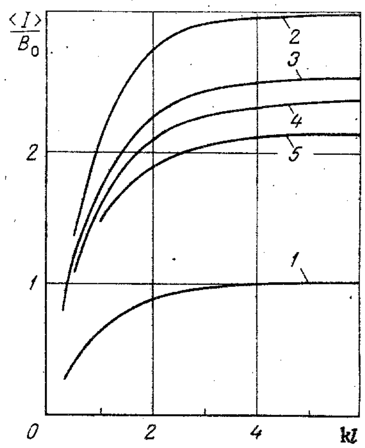

Fig. 1. Radiation of turbulent layer as a function of the optical thickness kl: 1) disregarding pulsations; 2) for optically thin pulsations [1]; 3-5) calculation from Eq. (22) with kl = 0.3 (3), 0.5 (4), 1 (5).

For $\alpha_2 = 4$, $\sigma = 0.2$, $kl = 1$, the terms ignored in Eqs. (18) and (19) give a relative contribution ~0.1, decreasing with decrease in kl.

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